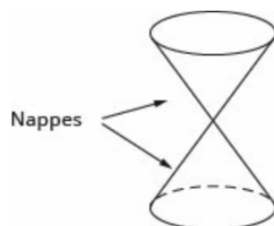# Conic Sections

In this chapter we will be looking at the conic sections, usually called the conics, and their properties. The conics are curves that result from a plane intersecting a double cone—two cones placed point-to-point. Each half of a double cone is called a nappe.

There are four conics—the **circle**, **parabola**, **ellipse**, and **hyperbola**. The next figure shows how the plane intersecting the double cone results in each curve.
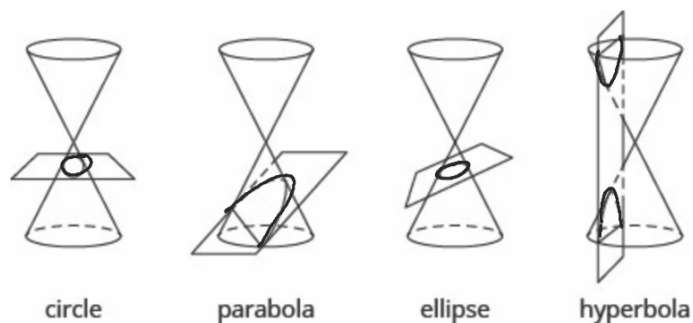
Each of the curves has many applications that affect your daily life, from your cell phone to acoustics and navigation systems. In this section we will look at the properties of a circle.

Distance Formula

The distance d between the two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance between the given points.

(−5, −3) and (7, 2)

$$d = \sqrt{(7-(-5))^2 + (2-(-3))^2}$$
$$\sqrt{12^2 + (5)^2}$$
$$\sqrt{144+25} = \sqrt{169} = 13$$

(−4, −5) and (5, 7)

$$d = \sqrt{(5-(-4))^2 + (7-(-5))^2}$$
$$= \sqrt{9^2 + 12^2}$$
$$= \sqrt{81 + 144} = \sqrt{225}$$
$$= 15$$

(−2, −5) and (−14, −10)

$$d = \sqrt{(-2-(-14))^2 + (-5-(-10))^2}$$
$$= \sqrt{(-12)^2 + (-5)^2}$$
$$= \sqrt{144 + 25}$$
$$\sqrt{169} = 13$$

MIDPOINT FORMULA

The midpoint of the line segment whose endpoints are the two points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

To find the midpoint of a line segment, we find the average of the x-coordinates and the average of the y-coordinates of the endpoints.

Find the midpoint of each segment given the endpoints.

(−5, −3) and (7, 2)

$$\left(\frac{-5+7}{2}, \quad \frac{-3+2}{2}\right)$$

$$\left(1, -\frac{1}{2}\right)$$

(−4, −5) and (5, 7)

$$\left(\frac{-4+5}{2}, \quad \frac{-5+7}{2}\right)$$
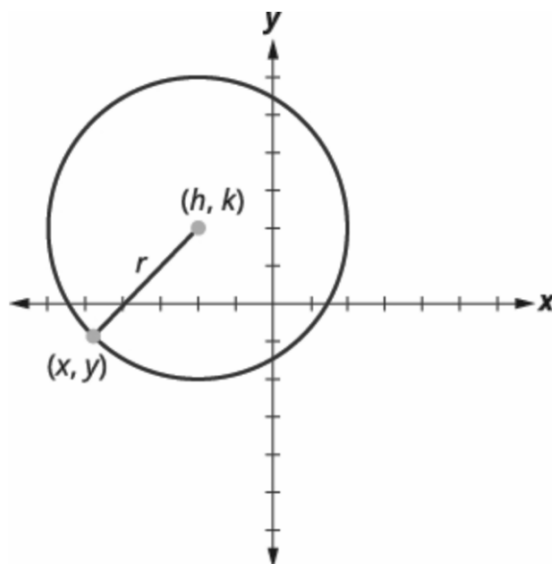
$$\left(\frac{1}{2}, 1\right)$$

(−2, −5) and (−14, −10)

$$\left(\frac{-2+-14}{2}, \quad \frac{-5+(-10)}{2}\right)$$
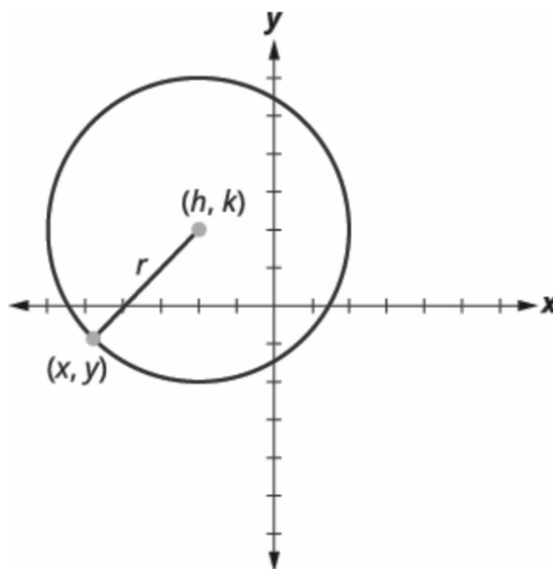
$$\left(-8, -\frac{15}{2}\right)$$

CIRCLE

A circle is all points in a plane that are a fixed distance from a given point in the plane. The given point is called the **center**, (h, k) , and the fixed distance is called the **radius**, r, of the circle.

We look at a circle in the rectangular coordinate system. The radius is the distance from the center, (h, k) , to a point on the circle, (x, y) .

|	
We look at a circle in the rectangular coordinate system. The radius is the distance from the center, (h, k), to a point on the circle, (x, y).	
To derive the equation of a circle, we can use the distance formula with the points (h, k), (x, y) and the distance, r.	$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
Substitute the values.	$$r = \sqrt{(x - h)^2 + (y - k)^2}$$
Square both sides.	$$r^2 = (x - h)^2 + (y - k)^2$$

STANDARD FORM OF THE EQUATION A CIRCLE

The standard form of the equation of a circle with center, (h, k), and radius, r, is
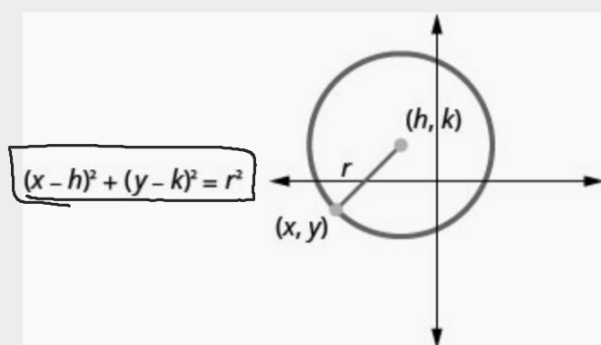
$$(x - h)^2 + (y - k)^2 = r^2$$

Write the standard form of the equation of the circle with radius 3 and center $(0, 0)$.

$$(0,0) = (h,k)$$

$$r^2 = 9$$

$$r = 3$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-0)^2 = 3^2$$

$$x^2 + y^2 = 9$$

Write the standard form of the equation of the circle with a radius of 6 and center $(0, 0)$.

$$x^2 + y^2 = 36$$

Write the standard form of the equation of the circle with radius 2 and center $(-1, 3)$.

$C = (-1, 3)$
       h   k

$(x-h)^2 + (y-k)^2 = r^2$

$(x+1)^2 + (y-3)^2 = 4$

Write the standard form of the equation of the circle with a radius of 7 and center $(2, -4)$.

$(x-2)^2 + (y+4)^2 = 49$

Write the standard form of the equation of the circle with a radius of 9 and center $(-3, -5)$.

$(x+3)^2 + (y+5)^2 = 81$

Write the standard form of the equation of the circle with center (2, 1) that also contains the point (−2, −2).

$x_1 y_1$

$x_2 y_2$

$$d = \sqrt{(2-(-2))^2 + (1-(-2))^2}$$

$$= \sqrt{(4)^2 + (3)^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25} = \boxed{5}$$

$$(x-2)^2 + (y-1)^2 = 25$$

Write the standard form of the equation of the circle with center (7, 1) that also contains the point (−1, −5).

$$d = \sqrt{(7-(-1))^2 + (1-(-5))^2}$$

$$= \sqrt{8^2 + 6^2}$$

$$= \sqrt{64 + 36}$$
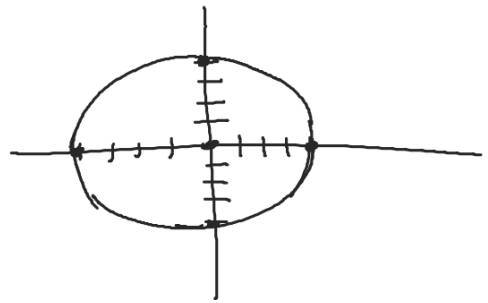
$$= \sqrt{100} = 10$$

$$(x-7)^2 + (y-1)^2 = 100$$

Find the center and radius and then graph the circle, $\dfrac{4x^2}{4} + \dfrac{4y^2}{4} = \dfrac{64}{4}$.
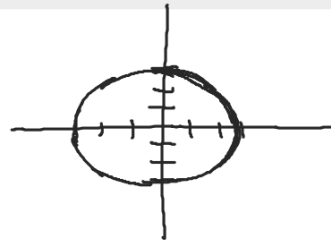
$$x^2 + y^2 = 16$$

Center $\qquad$ r = 4
$(0,0)$

@ Find the center and radius, then ⓑ graph the circle: $3x^2 + 3y^2 = 27$

$$x^2 + y^2 = 9$$

Center $\qquad$ r = 3
$(0,0)$

GENERAL FORM OF THE EQUATION OF A CIRCLE

The general form of the equation of a circle is

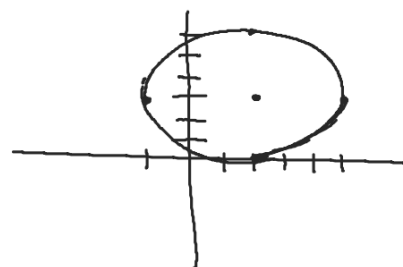$$x^2 + y^2 + ax + by + c = 0$$

ⓐ Find the center and radius, then ⓑ graph the circle: $x^2 + y^2 - 4x - 6y + 4 = 0$.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x^2 - 4x + 4) + (y^2 - 6y + 9) = -4 + 4 + 9$$

$$(x-2)^2 + (y-3)^2 = 9$$

Center $r = 3$
$(2,3)$

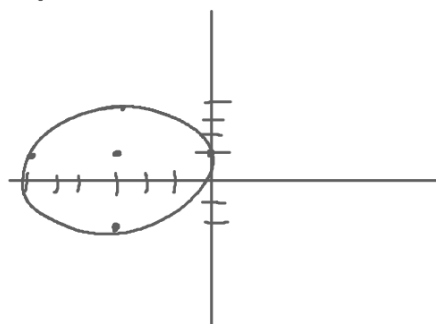ⓐ Find the center and radius, then ⓑ graph the circle: $x^2 + y^2 + 6x - 2y + 1 = 0$.

$(x^2 + 6x + 9) + (y^2 - 2y + 1) = -1 + 9 + 1$

$(x + 3)^2 + (y - 1)^2 = 9$

Center

$(-3, 1)$

$r = 3$

ⓐ Find the center and radius, then ⓑ graph the circle: $x^2 + y^2 + 8y = 0$.
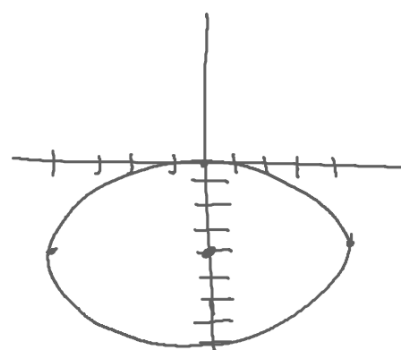
$$x^2 + \left(y^2 + 8y + 16\right) = 0 + 16$$

$$x^2 + (y+4)^2 = 16$$

$$(x-0)^2 + (y+4)^2 = 16$$

Center

$(0, -4)$

$r = 4$

@ Find the center and radius, then ⓑ graph the circle: $x^2 + y^2 - 2x - 3 = 0$.

$$(x^2 - 2x + 1) + y^2 = 3 + 1$$

$$(x-1)^2 + y^2 = 4$$

Center $= (1, 0)$

$r = 2$